

The distribution of annual incomes among all Americans is strongly skewed to the right (a few people make a lot of money). The mean of the distribution is \$67,000 and the standard deviation is \$40,000.

1) If many random samples of 500 Americans were taken, what would be the mean?

$$\mu_{\bar{x}} = \mu = \$67,000$$

2) What would be the standard deviation?

10% condition:  $10(500) = 5,000 \leq$  all Americans ✓

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{40,000}{\sqrt{500}} = \boxed{1,788.85}$$

3) Considering that the distribution of all incomes is strongly skewed to the right, is it possible to determine the probability that the mean of a single sample of 500 Americans would be less than \$65,000? Explain why. If it is possible determine the probability.

Yes → Central limit theorem says that if sample size (500) is greater than 30, then the sampling distribution is normal in shape, therefore we can use the normal distribution to find that probability.

$$P(\bar{x} < 65,000), N_{\bar{x}}(67,000, 1,788.85) \rightarrow \boxed{.1314}$$

$$\hookrightarrow z = \frac{65,000 - 67,000}{1,788.85} = -1.12 \rightarrow z\text{-table}$$

4) An educational study in the late 1990s found that small high schools were more commonly listed among the highest performing schools than large high schools. For a time this led to a push to create smaller schools. Then it was noted that small high schools also were more commonly listed among the worst performing schools. Explain why small high schools would be more likely to be rated high or low performing than large high schools. (Hint: Assume students are randomly assigned to a high school)

\* When considering sampling distributions, the larger the sample size the less variability in the desired statistic, so the closer the sample proportion/mean will be to the true mean/proportion. This means the ~~more likely~~ the larger the sample (in this case, school) the ~~more likely~~ less likely it will be among the more extreme values in the sampling distribution.

(larger sample = smaller variability / standard deviation of sampling distribution).



The weight of adult males in the U.S. has a mean of 172 pounds with a standard deviation of 29 pounds. *let  $\bar{x}$  = average weight of males in SRS of 46.*

5) Can you determine the probability of randomly selecting one male and finding his weight is over 180 pounds? Explain why or why not. If you can, what is the probability?

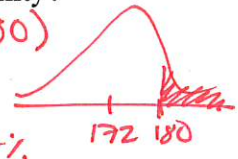
*You can not do this, as it is not stated that the population distribution is normal and the sample size of 1 is not greater than 30, so the CLT does not apply.*

6) Can you determine the probability of randomly selecting 46 males and finding the mean of their weights is over 180 pounds? Explain why or why not. If you can, what is the probability?

*Yes  $\rightarrow n = 46 \geq 30$ , so CLT applies  $\rightarrow N_{\bar{x}}(172, 4.276)$ ,  $P(\bar{x} > 180)$*

*10% condition:  $10(46) \leq$  all males  $\checkmark$   
 $\mu_{\bar{x}} = 172$   
 $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{29}{\sqrt{46}} = 4.276$*

*$z = \frac{180 - 172}{4.276} = 1.87$   $\rightarrow$  z-table  $.9693$*



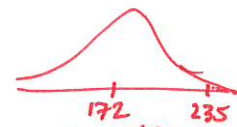
*There is about a 3.07% chance that a SRS of size 46 will have an average weight ~~less~~ greater than 180 lbs.*

7) The mean weight of the 46 players on the San Francisco 49ers is 235 pounds. Is this a representative sample for question 6? Why or why not?

*Given  $N_{\bar{x}}(172, 4.276)$  find  $P(\bar{x} \geq 235)$*

*$z = \frac{235 - 172}{4.276} = 14.733$*

*$\rightarrow$  z-table  $\rightarrow 1 - 1 = 0$*



*Not only is men on the 49ers not randomly selected but if they were there is an extremely low chance of getting an average weight of 235 or larger*

Records indicate that 24% of the students at Leland High School received at least one D or F last year. A random sample of 100 students was taken and students were asked if they received a D or F last year, only 15 of these students said they did.

*let  $\hat{p}$  = sample proportion of students in SRS of 100 who received D/F last year.*

*in a SRS of 46 men.*

8) What is the mean and standard deviation of the sampling distribution? Justify any calculations.

*10% condition:  $10(100) \leq$  all Leland HS students*

*$\mu_{\hat{p}} = p = .24$*

*$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.24(.76)}{100}} = .0427$*

9) What is the probability that 15 or fewer students would have a D or F in a 100 student sample? Justify any calculations.

*Normal condition:*

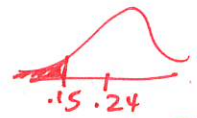
*$np = 100(.24) = 24 \geq 10 \checkmark$*

*$n(1-p) = 100(.76) = 76 \geq 10 \checkmark$*

*$\rightarrow N_{\hat{p}}(.24, .0427)$  find  $P(\hat{p} \leq .15)$*

*$z = \frac{.15 - .24}{.0427} = -2.10$*

*$\rightarrow$  z-table  $.0174$*



*There is about a 1.74% chance that a SRS of 100 Leland students would have 15 or fewer get a D/F last year.*

10) Do you believe the students in the sample were being truthful about their grades? Explain why or why not.

*No  $\rightarrow$  the probability of randomly selecting a sample and having the students tell the truth is less than 5%, the level of most statistically significant findings.*

*15 or fewer get a D/F last year.*